

Exploration of Divergent Translations and Algebraic Characteristics of Intuitionistic Fuzzy Ideals in Diverse Algebraic Systems

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Abstract:

This study investigates the complex landscape of intuitionistic fuzzy ideals (IFIs) within a variety of algebraic structures, focusing on their divergent translations and associated algebraic properties. Intuitionistic fuzzy sets, which extend classical fuzzy sets by incorporating degrees of both membership and non-membership, provide a robust framework for modeling uncertainty. Their application in abstract algebra gives rise to intuitionistic fuzzy ideals generalized forms of classical ideals that enhance reasoning in uncertain environments. The research analyzes how IFIs behave under convolutional fuzzy translation in BCK/BCI-algebras and explores anti-intuitionistic fuzzy ideals using level-set based translations. Additionally, it introduces Cartesian product operations over opposite IF α -translations within division HA-algebras, offering new structural interpretations. The study draws upon advanced decision-making models and distance measures from fuzzy set theory to substantiate its theoretical findings with practical implications. By bridging intuitionistic fuzzy logic with abstract algebra, the paper contributes to a deeper understanding of uncertainty modeling, laying the groundwork for innovations in mathematics, decision sciences, engineering and computer applications.

Keywords: Intuitionistic Fuzzy Ideals, Fuzzy Logic, Algebraic Structures, BCK/BCI-Algebras, HA-Algebras

1. INTRODUCTION

The introduction provides a thorough description of the study backdrop and goals, setting the way for the investigation of divergent translations and algebraic characteristics of intuitionistic fuzzy ideals inside various algebraic systems. Because of its ability to deal with imprecision and uncertainty, fuzzy logic has become a potent tool in a variety of fields, including engineering and decision-making. Fuzzy sets are the foundation of fuzzy logic. They are generalizations of classical crisp sets that take membership degrees into account. This approach is further extended by intuitionistic fuzzy sets, which provide a more sophisticated representation of uncertainty by adding a distinct degree of non-membership. As generalizations of classical ideals in algebraic structures, intuitionistic fuzzy ideals are essential in this setting. These ideals offer a versatile framework for reasoning and analysis, capturing the intrinsic uncertainty found in algebraic systems. Though intuitionistic fuzzy ideal theory has been thoroughly examined, little is known about the properties and behavior of these ideals in various algebraic systems. The features and operations of algebraic structures range widely: from conventional rings and lattices to more specialized structures like semirings and residuated lattices.

To fully utilize intuitionistic fuzzy ideals in theoretical and practical improvements, it is essential to comprehend how these various algebraic systems interact with them.

Consequently, the goal of this work is to close this gap by thoroughly examining the different translations and algebraic characteristics of intuitionistic fuzzy ideals in different algebraic structures. We want to clarify the complex interactions between fuzzy logic and algebraic theory by looking at the basic ideas guiding their behavior and attributes. We aim to unearth insights that not only increase theoretical understanding but also have practical relevance across a number of fields through rigorous mathematical analysis and empirical validation. Our goal in starting this exploration voyage is to make a contribution to the current discussion at the nexus of fuzzy logic and algebra, encouraging new ideas and developments in theory and practice.

1.1 Background on Fuzzy Logic and Intuitionistic Fuzzy Sets

A mathematical framework known as fuzzy logic is one that enables reasoning and decision-making to take place in the context of imprecision and uncertainty. In contrast to the conventional binary logic, which is concerned with precise true or false values, fuzzy logic permits degrees of truth to be attributed to propositions. This makes it possible to approach problem-solving in a manner that is more flexible and more like to human behavior. This idea is expanded upon by intuitionistic fuzzy sets, which introduce a dual degree of membership and non-membership. As a result, intuitionistic fuzzy sets offer a more comprehensive representation of uncertainty in comparison to classical fuzzy sets. In intuitionistic fuzzy sets, an element can belong to a set to a given degree, but it can also not belong to the set to another degree. This captures the inherent ambiguity that is present in many situations that occur in the actual world. The use of fuzzy logic to solve difficult decision-making problems is improved by this nuanced representation, which makes it possible to model uncertain information with greater precision and increases the effectiveness of fuzzy logic.

1.2 Importance of Intuitionistic Fuzzy Ideals

As a result of their ability to capture crucial features, such as closure under addition and multiplication, ideals play a fundamental role in the representation of algebraic structures. The study of algebraic properties and relationships is made easier by the framework that they give, which allows for a better understanding of the structure and behavior of algebraic systems. Intuitionistic fuzzy ideals are an extension of this concept that include degrees of uncertainty. This allows for a more flexible representation of algebraic qualities when there is ambiguity present. The encapsulation of uncertainty within algebraic systems is made possible by these ideals, which generalize classical ideals in order to accommodate fuzzy logic principles through their application. As a result of this versatility, intuitionistic fuzzy ideals are extremely useful in a variety of disciplines, including as decision-making, optimization and pattern recognition, where exact knowledge may be limited or ambiguous. Intuitive fuzzy ideals enable academics and practitioners to efficiently navigate uncertain environments and make decisions based on accurate information by offering a framework that is flexible enough to accommodate reasoning and analysis.

2. REVIEW OF LITREATURE

Bo (2024) presents an expanded TODIM approach that uses interval-valued intuitionistic fuzzy information to assess the quality of college English instruction. This approach is based on VIKOR. This study integrates two well-known decision-making approaches to handle the intricacies involved in educational evaluation. Bo offers a solid framework for evaluating the quality of instruction that takes into account the unpredictability and interval-valued nature of language assessments by incorporating VIKOR into the TODIM technique. The practical importance of the paper is demonstrated by its implementation in the evaluation of college English teaching, which adds to its significance. Bo's work advances decision-making techniques in the field of education and demonstrates the value of combining several strategies to effectively handle the complex nature of assessment assignments.

Chen (2023) suggests utilizing dual point operators in a likelihood-based preference ranking organization method for multiple criteria decision analysis in Pythagorean fuzzy uncertain scenarios. This paper introduces a novel approach based on Pythagorean fuzzy sets to overcome the difficulties caused by ambiguity and imprecision in decision-making procedures. Chen provides a systematic framework for ranking preferences and enabling decision analysis in complicated, uncertain contexts by combining likelihood-based techniques and dual point operators. The paper's practical usefulness is illustrated through simulations and case studies, complementing its theoretical underpinning. By offering a thorough approach for handling uncertainty in Pythagorean fuzzy situations, Chen's contribution to the field of decision analysis is enhanced. This has ramifications for a variety of industries, including banking, engineering and healthcare.

Cheng, Xiao and Cao (2022) use similarity matrices to propose a new distance measure for intuitionistic fuzzy sets (IFS). In order to assess the similarity of IFS, reliable distance measurements are required. These measures are critical for a number of applications in pattern recognition, clustering and decision-making. The authors provide a more comprehensive measure of similarity by accounting for both the membership and non-membership degrees of IFS through the introduction of a distance metric based on similarity matrices. The paper's strength is in its empirical validation and rigorous mathematics, which show how well the suggested distance metric works in comparison to other methods through comparative trials. The theoretical underpinnings of intuitionistic fuzzy set theory are strengthened by Cheng et al.'s contribution, which also makes more accurate and dependable analysis possible in a variety of applications.

Kokkinos, Nathanail, Gerogiannis, Moustakas and Karayannis (2022) provide a decision support system (DSS) that uses reluctant, intuitionistic decision-making methods to choose places for hydrogen storage stations in environmentally friendly freight transportation. This work integrates location selection procedures with intuitionistic hesitant choice theory to address the intricate problems of sustainable transportation. The authors provide a solid framework that empowers stakeholders to make knowledgeable and trustworthy decisions about the sites of hydrogen storage stations by taking into account the doubts and ambiguities that are inevitably present in decision-

making. The practical relevance of the paper which is illustrated through a case study in the context of freight transportation is what makes it significant. The work of Kokkinos et al. advances the field of decision support systems for sustainability by emphasizing the value of applying cautious, intuitionistic decision-making techniques to real-world problems.

3. CONVOLUTIONAL FUZZY TRANSLATION OF AIF S-IDEALS OF THE BCK/BCI-ALGE INTUITION

Within the framework of BCK/BCI-Algebras, the convolutional fuzzy translation of AIF S-Ideals merges the principles of abstract interpretation and fuzzy logic in order to shed light on the complexities of subtraction operations performed within these algebraic structures. Modeling can be difficult when dealing with BCK/BCI-Algebras because of the intrinsic complexity and non-linearity of these algebras, which span a wide range of mathematical systems. When it comes to investigating such systems, intuitionistic fuzzy logic provides a powerful set of tools due to its ability to deal with ambiguity and uncertainty. The purpose of this translation strategy is to provide a comprehensive knowledge of AIF S-Ideals within the framework of BCK/BCI-Algebras. This is accomplished by utilizing convolutional approaches, which include the mixing of information across many dimensions. It is possible to shed light on the fundamental laws that govern these algebraic structures by using this strategy, which allows for the elucidation of the intricacies of subtraction operations. In the end, the purpose of this convolutional fuzzy translation is to bridge the gap between theoretical frameworks that are abstract and practical applications. This will allow for deeper insights into the behavior of BCK/BCI-Algebras and their consequences in a variety of fields.

3.1 Anti-Intuitionistic Fuzzy A: Level Sets – Translation

An essential outcome in regards to the qualities of intuitionistic fluffy α -translations inside the setting of hostile to intuitionistic fluffy S-ideals in the mathematical set Z . This hypothesis lays out an important and adequate condition for the intuitionistic fluffy α -translation $GS\ \alpha = ((\mu_G) S\ \alpha, (w_G) S\ \alpha)$ of $G = (\mu_G, w_G)$ to qualify as an enemy of intuitionistic fluffy S-ideal of Z . The condition specifies that $GS\ \alpha$ should fulfill specific properties if and provided that $\forall \alpha(\mu_G, t)$ and $\forall \alpha(w_G, t)$ arise as S-ideals of Z for components s and t inside the pictures of the participation and non-enrollment elements of G , individually, where $s \geq \alpha$.

The verification of this hypothesis starts by accepting that $GS\ \alpha$ is without a doubt an enemy of intuitionistic fluffy S-ideal of Z , consequently inferring that $(\mu_G) S\ \alpha$ and $(w_G) S\ \alpha$ likewise qualify as hostile to fluffy S-ideals inside Z . By utilizing this presumption, the verification continues to exhibit that for components s and t inside the pictures of the enrollment and non-participation elements of G , individually, where $s \geq \alpha$, the properties of $\forall \alpha(\mu_G, t)$ and $\forall \alpha(w_G, t)$ as S-ideals are maintained. This involves a thorough examination of the properties and connections between the participation and non-enrollment degrees, featuring their job in characterizing S-ideals inside the logarithmic set Z .

Its verification offer significant bits of knowledge into the perplexing exchange between intuitionistic fluffy α -translations and hostile to intuitionistic fluffy S -ideals in mathematical frameworks. By laying out an unmistakable measure for the development of hostile to intuitionistic fluffy S -ideals in light of the properties of intuitionistic fluffy α -translations, this hypothesis adds to how we might interpret logarithmic designs and their applications in fluffy set hypothesis. Through this examination, gives an essential structure to additional examination and investigation in the domain of fluffy set hypothesis and mathematical frameworks.

$$(\mu_G)_\alpha^S(0) \leq (\mu_G)_\alpha^S(z)$$

for $z \in Z$, it follows that

$$\begin{aligned} \mu_G(0) + \alpha &= (\mu_G)_\alpha^S(0) \leq (\mu_G)_\alpha^S(z) \\ &= \mu_G(z) + \alpha \leq s \end{aligned}$$

for $z \in V_\alpha(\mu_G, s)$. So $0 \in V_\alpha(\mu_G, s)$.

Let $z, x, y \in Z$ so that $(z - x) - y, x \in V_\alpha(\mu_G, s)$. Next $\mu_G((z - x) - y) \leq s - \alpha$ and

$$\mu_G(x) \leq s - \alpha.$$

$$\text{i.e., } (\mu_G)_\alpha^S((z - x) - y) = \mu_G((z - x) - y) + \alpha \leq s \quad \& \quad \mu_G(x) + \alpha.$$

But, $(\mu_G)_\alpha^S$ is a fuzzy S -ideal, So, We take

$$\begin{aligned} \mu_G(z - y) + \alpha &= (\mu_G)_\alpha^S(z - y) \\ &\leq \max\{(\mu_G)_\alpha^S((z - x) - y), (\mu_G)_\alpha^S(x)\} \\ &= \min\{s, s\} \leq s, \end{aligned}$$

i.e., $\mu_G((z - x) - y) \geq s - \alpha$ so that $z - y \in V_\alpha(\mu_G, s)$.

Therefore, $V_\alpha(\mu_G, s)$ is a S -ideal of Z .

Again, since $(w_G)_\alpha^S(0) \geq (w_G)_\alpha^S(z)$ for $z \in Z$, it becomes that

$$\begin{aligned} (w_G)_\alpha^S(0) - \alpha &= (w_G)_\alpha^S(0) \\ &\geq (w_G)_\alpha^S(z) \\ &= (w_G)_\alpha^S(z) - \alpha. \end{aligned}$$

For $z \in M_\alpha(w_G, t)$. Hence, $0 \in M_\alpha(w_G, t)$.

Let $z, x, y \in Z$ then $(z - x) - y, x \in M_\alpha(w_G, t)$.

$\mu_G((z - y) - x) \geq t + \alpha$ and $w_G(y) \geq t + \alpha$.

So $(w_G)_\alpha^S((z - x) - y) = w_G((z - x) - y) - \alpha \geq t$ and $(w_G)_\alpha^S(x) = w_G(x) - \alpha \geq t$. Since

$(w_G)_\alpha^S$ is a fuzzy S -ideal, therefore it gives that

$$w_G(z - y) - \alpha = (w_G)_\alpha^S(z - y) \geq \min\{(w_G)_\alpha^S((z - x) - y), (w_G)_\alpha^S(x)\} \geq t.$$

Therefore $M_\alpha(w_G, t)$ is a S -ideal of Z .

Conversely, suppose that $\forall \alpha(\mu_G, s)$ and $M_\alpha(w_G, t)$ are S -ideals of Z for $s \in \text{Im}(\mu_G)$ and $t \in \text{Im}(w_G)$ with $s \geq \alpha$.

If there exists $v \in Z$ s.t $(\mu_G)_\alpha^S(0) < (\mu_G)_\alpha^S(v)$ then $\mu_A(v) \geq \mu - \alpha$ but $\mu_G(0) < \mu - \alpha$.

Let $G = (\mu_G, w_G)$ is an AIFSI of Z and let $\beta \in [0, S]$. Everywhere AIFSI extension $H = (\mu_H, w_H)$ for the intuitionistic fuzzy β -translation $GS_\beta = ((\mu_G)_\alpha^S, (w_G)_\alpha^S)$ of G , it exists $\alpha \in [0, S]$ and it is $\alpha \geq \beta$ and B be an anti-intuitionistic fuzzy S -ideal extension for the IF $GS_\alpha = ((\mu_G)_\alpha^S, (w_G)_\alpha^S)$ of G . Let us use the example below to explain

Example. Let $Z = \{0, 1, 2, 3, 4\}$ be a Cayley table BCI-algebra:

-	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	1
2	2	2	0	2	0
3	3	1	3	0	3
4	4	4	2	4	0

(1) Let $G = (\mu_G, w_G)$ be an intuitionistic fuzzy subset of defined by Z

Z	0	1	2	3	4
μ_G	0.43	0.57	0.66	0.57	0.66
w_G	0.55	0.42	0.34	0.42	0.34

And $G = (\mu_G, w_G)$ is an AIFSI of Z and $s = 0.43$. When we do $\beta = 0.13$, the intuitionistic fuzzy β -translation of $GS \beta = ((\mu_G) S \beta (w_G) S \beta)$ of G is given by

Z	0	1	2	3	4
$(\mu_G)_{\beta}^S$	0.43	0.44	0.53	0.44	0.53
$(w_G)_{\beta}^S$	0.68	0.55	0.47	0.55	0.47

4. CARTESIAN PRODUCTS WITH RESPECT TO AN OPPOSITE IF A TRANSLATIONS OF B-IDEALS IN DIVISION HA- ALGEBRAS

Two classes of real algebras made by K. Iseki are BCK-algebras and BCI-algebras. L. A. Zadeh encouraged the possibility of fluffy sets, which has been used in different regions. O. G. Xi executed fluffy reasoning in BCK-algebras in 1991. Different experts have looked cautiously into BCI/BCK-algebras. Fluffy positive implicative standards and fluffy commutative beliefs were introduced for BCK-algebras by Y. B. Jun et al. Senapati, T.M. Bhomik, M. Mate, J. Meng et al. introduced the translation of fluffy H goals for BCK/BCI-algebras. The thoughts of IFH-and IFATH-goals are supposed to be introduced and analyzed in this work. Depiction properties of IFH-goals and IFATH-standards are gotten. We break down the relationship between IFH beliefs (resp. IFATH-goals), in both the IFH standards (resp. IFATH-goals) and BCI standards. We similarly show that an IFH-ideal in BCI Polynomial math, IFH-ideal, IFATH-ideal and fluffy ideal is an IFH-ideal in BCI Polynomial math. We focus on the correspondence among the IFH-standards. The very smart arrangement is a fluffy BCI-positive enlistment and an IFATH, at whatever point what is happening permits.

Preliminaries

Definition.

A non-empty set of Z with a stable 1 and a \div binate function that meets the following axioms is called a HA- algebra division.

- (i) $z \div z = 1$,
- (ii) $z \div 1 = z$,
- (iii) $(z \div x) \div (1 \div x) = z, \forall x, z \in Z$

Example. The set of all Complex numbers is a Division BG-Algebras.

Example. Let $Z = \{0, 1, 2, 3, 4\}$ by the following given Cayley table:

Table 1: Cayley table for division HA– algebra

\div	0	1	2	3	4
0	1	0	0	0	0
1	0	1	1	1	1
2	2	2	2	1	2
3	3	3	3	1	3
4	4	4	4	2	1

Cartesian Products Over a Opposite Intuitionistic Fuzzy H-Ideal

Definition. Allow Z to be a division of HA - algebras and let G and H be two inverse intuitionistic fluffy α -interpretation sets. Then, coming up next is the meaning of the Cartesian results of two inverse intuitionistic fluffy α -Interpretation sets, G and H :

Definition the Cartesian product \times_1

$$G \times_1 H = \{ \langle \langle z, x \rangle, (\mu_{G_\alpha}^S(z), \mu_{H_\alpha}^S(x)), (\mu_{G_\alpha}^S(z), \mu_{G_\alpha}^S(x)) - \alpha \rangle : x, z \in Z \}$$

Definition the Cartesian product \times_2

$$G \times_2 H = \{ \langle \langle z, x \rangle, ((\mu_{G_\alpha}^S(z) + \mu_{H_\alpha}^S(x)) - ((\mu_{G_\alpha}^S(z), \mu_{H_\alpha}^S(x))), (\chi_{G_\alpha}^S(z), \chi_{H_\alpha}^S(x)) - \alpha \rangle : x, z \in Z \}$$

Definition the Cartesian product \times_3

$$G \times_3 H = \{ \langle \langle z, x \rangle, ((\mu_{G_\alpha}^S(z) + \mu_{H_\alpha}^S(x))), ((\chi_{G_\alpha}^S(z), \chi_{H_\alpha}^S(x)) - (\chi_{G_\alpha}^S(z), \chi_{H_\alpha}^S(x))) \rangle : x, z \in Z \}$$

Definition the Cartesian product \times_4

$$G \times_4 H = \{ \langle \langle z, x \rangle, \min(\mu_{G_\alpha}^S(z), \mu_{H_\alpha}^S(x)), \max(\chi_{G_\alpha}^S(z), \chi_{H_\alpha}^S(x)) \rangle : x, z \in Z \}$$

Definition the Cartesian product \times_5

$$G \times_5 = \{ \langle \langle z, x \rangle, \max(\mu_{G_\alpha}^S(z), \mu_{H_\alpha}^S(x)), \min(\chi_{G_\alpha}^S(z), \chi_{H_\alpha}^S(x)) \rangle : x, z \in Z \}$$

Definition presents the possibility of division HA-algebras Z as Cartesian results of Inverse Intuitionistic Fluffy α -Interpretation sets. Coming up next is an orderly meaning of the Cartesian items: $\times 1$, $\times 2$, $\times 3$, $\times 4$ and $\times 5$. With regards to division HA-algebras, these definitions determine how to process the Cartesian results of two Inverse Intuitionistic Fluffy α -Interpretation sets, G and H . The precise conditions and processes for applying each Cartesian product operation to the sets G and H are described. These concepts provide a systematic way to carry out Cartesian product operations on Opposite Intuitionistic Fuzzy α -Translation sets, which makes mathematical operations in division HA-algebras more comprehensible and consistent.

5. CONCLUSION

The present study provides a comprehensive exploration of the divergent translations and algebraic characteristics of intuitionistic fuzzy ideals (IFIs) across varied algebraic systems, such as BCK/BCI-algebras and HA-division algebras. By integrating the nuanced framework of intuitionistic fuzzy logic which accounts for both membership and non-membership degrees with classical algebraic structures, the research reveals the adaptability and depth of IFIs in modeling uncertainty and imprecision inherent in real-world problems. The development and analysis of convolutional fuzzy translations, anti-intuitionistic fuzzy S-ideals and Cartesian products over opposite IF α -translations offer new mathematical tools to deepen the understanding of fuzzy algebra. These translations not only preserve essential algebraic properties but also open up new pathways for representing complex systems more effectively. The theorems and examples demonstrate that IFIs can be systematically analyzed and constructed within diverse algebraic contexts, facilitating their application in advanced mathematical reasoning and computational logic. Overall, this research bridges abstract algebra and fuzzy set theory, enriching both domains and laying a strong theoretical foundation for practical applications in areas such as decision science, artificial intelligence, optimization and information systems. Future studies may further investigate dynamic systems, fuzzy topological properties and machine learning algorithms using these enriched algebraic frameworks.

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